

Lecture 3

Friday, January 15, 2021 2:08 PM

* Prayer

* Spiritual thoughts :

- The joy of learning is part of our experience in mortality (Men are that they may have joy.
- Record your experience when you realize that what you are learning confirms a Gospel principle.

* Answering questions ...

Dot product

$$a = \langle a_1, a_2, a_3 \rangle$$

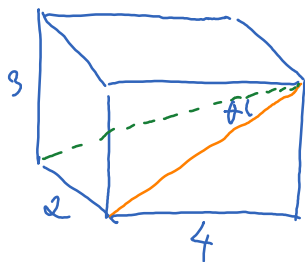
$$b = \langle b_1, b_2, b_3 \rangle$$

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

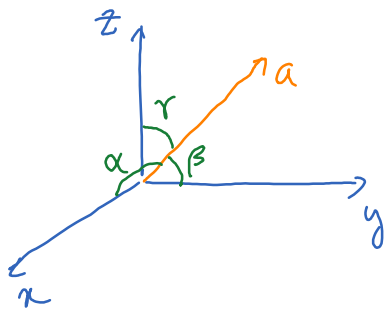
Angle

$$\cos \theta = \frac{a \cdot b}{|a||b|} \quad (\text{Law of Cosine of a Triangle})$$

Ex



$$\theta = ?$$



$$\cos \alpha = \frac{\vec{i} \cdot \vec{a}}{|\vec{i}| |\vec{a}|} = \frac{\vec{i} \cdot \vec{a}}{|\vec{a}|} = \vec{i} \cdot \frac{\vec{a}}{|\vec{a}|}$$

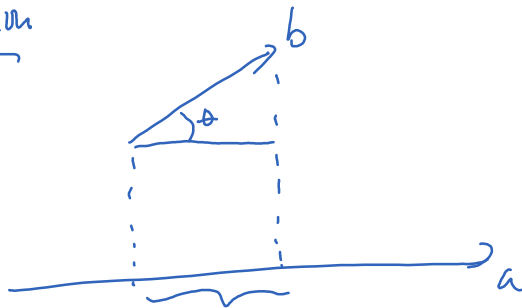
$$\cos \beta = \vec{j} \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$\cos \gamma = \vec{k} \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$\frac{\vec{a}}{|\vec{a}|} = \left\langle \cos \alpha, \cos \beta, \cos \gamma \right\rangle$$

$\uparrow \quad \uparrow \quad \uparrow$
 direction cosine

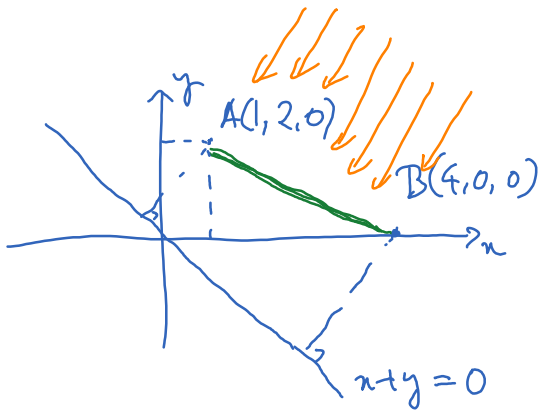
Projection



$$\begin{aligned} \text{Comp}_a b &= |b| \cos \theta \\ &= |b| \frac{a \cdot b}{|a| |b|} = \frac{a \cdot b}{|a|} \end{aligned}$$

$$\text{proj}_a b = (\text{Comp}_a b) \frac{a}{|a|} = \frac{a \cdot b}{|a|^2} a$$

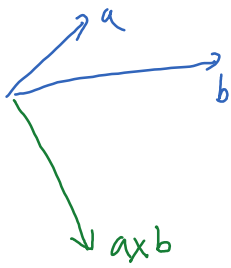
Ex
 discussion time



What is the length of the shadow?

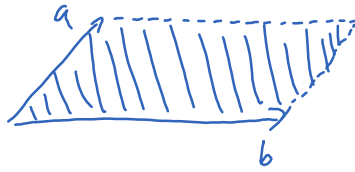
Cross product

Dot product of two vectors is a number.
 Cross product of two vectors is a vector.



$a \times b$ is perpendicular to both a and b , directed by right hand rule.

$|a \times b| =$ parallelogram formed by a and b .



Algebraically, to find $a \times b$, one does the following

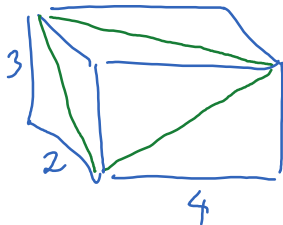
$$a = \langle a_1, a_2, a_3 \rangle$$

$$b = \langle b_1, b_2, b_3 \rangle$$

$$\begin{array}{cccccc} a_1 & a_2 & a_3 & a_1 & a_2 & \\ & \diagdown & \diagup & \diagdown & \diagup & \\ & b_2 & b_3 & b_1 & b_2 & \\ b_1 & & & & & \end{array}$$

$$a \times b = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

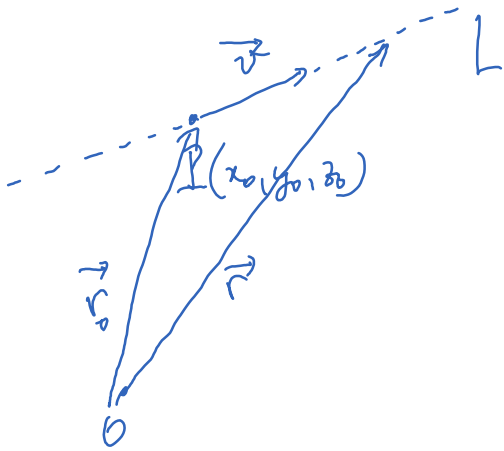
Ex: find the area of the green triangle.



Ex: $a \times b \stackrel{?}{=} b \times a$

$(a \times b) \times c \stackrel{?}{=} a \times (b \times c)$

Equation of lines



$$\vec{r} = \langle x, y, z \rangle$$

$\vec{r} - \vec{r}_0$ is parallel to \vec{v} .

$$\vec{r} - \vec{r}_0 = t\vec{v}$$

$$\boxed{\vec{r} = \vec{r}_0 + t\vec{v}} \quad \text{vector equation}$$

$$x = x_0 + ta$$

$$y = y_0 + tb$$

$$z = z_0 + tc$$

} parametric equations

11/11

